# Two-step deterministic remote preparation of an arbitrary quantum state in the whole Hilbert space

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We present a two-step exact remote state preparation protocol of an arbitrary qubit with the aid of a three-particle Greenberger-Horne-Zeilinger state. Generalization of this protocol for higher-dimensional Hilbert space systems among three parties is also given. We show that only single-particle von Neumann measurement, local operation and classical communication are necessary. Moreover, since the overall information of the quantum state can be divided into two different parts, which may be at different locations, this protocol may be useful in the quantum information field.

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## I. INTRODUCTION

Quantum information theory has produced many interesting and important developments that are not possible classically in recent years, in which quantum entanglement and classical communication are two elementary resources. Two surprising discoveries in this area are teleportation and remote state preparation (RSP). Quantum teleportation process, originally proposed by Bennett et al [1], can transmit an unknown quantum state from a sender (called Alice) to a spatially distant receiver (called Bob) via a quantum channel with the help of some classical information. Recently, Lo [2], Pati [3] and Bennett et al [4] have presented an interesting application of quantum entanglement, i.e., remote state preparation that correlates closely to teleportation. RSP is called "teleportation of a known quantum state", which means Alice knows the precise state that she will transmit to Bob. Her task is to help Bob construct a state that is unknown to him by means of a prior shared entanglement and a classical communication channel. So the goal of RSP is the same as that of quantum teleportation. The main difference between RSP and teleportation is that in the former Alice is assumed to know completely the state to be prepared remotely by Bob; in particular, Alice need not own the state, but only know information about the state, while in the latter Alice must own the transmitted state, but neither she nor Bob has knowledge of the transmitted state.

So far, RSP has attracted much attention [5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. There are many kinds of RSP methods in theory, such as low-entanglement RSP [5], higher-dimension RSP [6], optimal RSP [7], oblivious RSP [8], RSP without oblivious conditions [9], RSP for multiparties [10], and continuous variable RSP in phase space[11, 12], etc. On the other hand, some RSP schemes have been implemented experimentally with the technique of NMR [16] and spontaneous parametric down-conversion [17, 18]. In addition, some authors have also investigated the RSP protocol using different quantum channels such as partial EPR pairs [19] and three-particle Greenberger-Horne-Zeilinger (GHZ) state [20]. To our best knowledge, up to now there is no RSP protocol which determinately generate an arbitrary qubit with unit success probability. They mainly concentrate on RSP of some special ensembles of a quantum state. For example, some schemes discuss how to successfully remotely prepare the state in subspace of the whole real Hilbert space or chosen from equatorial line on Bloch sphere.

In this paper, we propose a two-step deterministic RSP protocol via previously shared entanglement, a single-particle von Neumann measurement, local operation and classical communication. Generalization of this protocol for higher-dimensional Hilbert space systems among three parties is also presented. We will see that the overall information of an arbitrary quantum state can be divided into two different parts. They are expressed by $\theta$ and $\varphi$ respectively, which may be at different locations. So this protocol may be useful in the quantum information field, such as quantum state sharing, converging the split information at one point, etc.

## II. DETERMINISTIC RSP OF AN ARBITRARY QUBIT USING A GHZ STATE AS A QUANTUM CHANNEL

Let us consider a pure state $|\psi\rangle \in H = C^2$ which is the state of a qubit. An arbitrary qubit can be represented as

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \tag{1}$$

where we can choose $\alpha$ to be real and $\beta$ to be complex number and $|\alpha|^2+|\beta|^2 = 1$. This qubit can be represented by a point on the unit two-dimensional sphere, known as

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Bloch sphere, with the help of two real parameters $\theta$ and $\varphi$. So we can rewrite Eq.(1) as

$$|\psi\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)e^{i\varphi}|1\rangle. \tag{2}$$

Now Alice wants to transmit the above qubit to Bob. The quantum channel shared by Alice and Bob is the three-particle GHZ state

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{123}. \tag{3}$$

The particles 1 and 2 belong to Alice and the particle 3 is held by Bob. As a matter of fact, the state $|\Phi\rangle$ can be easily generated from the Bell state $\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{23}$, because particles 1 and 2 belong to Alice. A Controlled-Not gate can transform $\frac{1}{\sqrt{2}}|0\rangle_1(|00\rangle + |11\rangle)_{23}$ into $|\Phi\rangle$, when particle 2 and particle 1 are a controlled qubit and a target qubit, respectively. We suppose the qubit $|\psi\rangle$ is known to Alice, i.e. Alice knows $\theta$ and $\varphi$ completely, but Bob does not know them at all. Since Alice knows the state she can choose to measure the particles 1 and 2 in any basis she wants. First, Alice performs a projective measurement on particle 1. The measurement basis chosen by Alice is a set of mutually orthogonal basis vectors $\{|\phi\rangle, |\phi_\perp\rangle\}$, which is related to the computation basis $\{|0\rangle, |1\rangle\}$ in the following manner

$$\begin{aligned} |\phi\rangle_1 &= \cos(\theta/2)|0\rangle_1 + \sin(\theta/2)|1\rangle_1, \\ |\phi_\perp\rangle_1 &= \sin(\theta/2)|0\rangle_1 - \cos(\theta/2)|1\rangle_1. \end{aligned} \tag{4}$$

By this change of basis, the normalization and orthogonality relation between basis vectors are preserved. Using Eq.(4), we can express Eq.(3) as

$$|\Phi\rangle = \frac{1}{\sqrt{2}}(|\phi\rangle_1|\Psi\rangle_{23} + |\phi_\perp\rangle_1|\Psi_\perp\rangle_{23}), \tag{5}$$

where

$$\begin{aligned} |\Psi\rangle_{23} &= \cos(\theta/2)|00\rangle_{23} + \sin(\theta/2)|11\rangle_{23}, \\ |\Psi_\perp\rangle_{23} &= \sin(\theta/2)|00\rangle_{23} - \cos(\theta/2)|11\rangle_{23}. \end{aligned} \tag{6}$$

Now Alice measures the particle 1. For example, if Alice's von Neumann measurement result is $|\phi\rangle_1$, then the state of particles 2 and 3, as shown by Eq.(5), will collapse into $|\Psi\rangle_{23}$. Next, Alice performs another projective measurement on particle 2. The measurement basis is also a set of mutually orthogonal basis vectors $\{|\eta\rangle, |\eta_\perp\rangle\}$, the relation between the measurement basis $\{|\eta\rangle, |\eta_\perp\rangle\}$ and the computation basis $\{|0\rangle, |1\rangle\}$ is given by

$$|\eta\rangle_2 = \frac{1}{\sqrt{2}}(|0\rangle_2 + e^{-i\varphi}|1\rangle_2), \;\; |\eta_\perp\rangle_2 = \frac{1}{\sqrt{2}}(|0\rangle_2 - e^{-i\varphi}|1\rangle_2). \tag{7}$$

Then, we have

$$|\Psi\rangle_{23} = \frac{1}{\sqrt{2}}(|\eta\rangle_2|\psi\rangle_3 + |\eta_\perp\rangle_2|\psi'\rangle_3), \tag{8}$$

TABLE I: Alice's measurement basis on particle 1 (MB1), Alice's measurement outcome for particle 1 (AMO1), Alice's measurement basis on particle 2 (MB2), Alice's measurement outcome for particle 2 (AMO2), the collapse states for particle 3 (CS3) and Bob's appropriate unitary operation (BAUO)

| MB1 | AMO1 | MB2 | AMO2 | CS3 | BAUO |
|---|---|---|---|---|---|
| $\{\|\phi\rangle, \|\phi_\perp\rangle\}$ | $\|\phi\rangle_1$ | $\{\|\eta\rangle, \|\eta_\perp\rangle\}$ | $\|\eta\rangle_2$ | $\cos(\theta/2)\|0\rangle+$ $\sin(\theta/2)e^{i\varphi}\|1\rangle$ | $I$ |
| $\{\|\phi\rangle, \|\phi_\perp\rangle\}$ | $\|\phi\rangle_1$ | $\{\|\eta\rangle, \|\eta_\perp\rangle\}$ | $\|\eta_\perp\rangle_2$ | $\cos(\theta/2)\|0\rangle-$ $\sin(\theta/2)e^{i\varphi}\|1\rangle$ | $\sigma_z$ |
| $\{\|\phi\rangle, \|\phi_\perp\rangle\}$ | $\|\phi_\perp\rangle_1$ | $\{\|\xi\rangle, \|\xi_\perp\rangle\}$ | $\|\xi\rangle_2$ | $\sin(\theta/2)e^{i\varphi}\|0\rangle-$ $\cos(\theta/2)\|1\rangle$ | $\sigma_z\sigma_x$ |
| $\{\|\phi\rangle, \|\phi_\perp\rangle\}$ | $\|\phi_\perp\rangle_1$ | $\{\|\xi\rangle, \|\xi_\perp\rangle\}$ | $\|\xi_\perp\rangle_2$ | $\sin(\theta/2)e^{i\varphi}\|0\rangle+$ $\cos(\theta/2)\|1\rangle$ | $\sigma_x$ |

where

$$\begin{aligned} |\psi\rangle &= \cos(\theta/2)|0\rangle + \sin(\theta/2)e^{i\varphi}|1\rangle, \\ |\psi'\rangle &= \cos(\theta/2)|0\rangle - \sin(\theta/2)e^{i\varphi}|1\rangle. \end{aligned} \tag{9}$$

If Alice's von Neumann measurement result is $|\eta\rangle_2$, the particle 3 can be found in the original state $|\psi\rangle$, which is nothing but the remote state preparation of the known qubit. If the outcome of Alice's measurement result is $|\eta_\perp\rangle_2$, then the classical communication from Alice will tell Bob that he has obtained a state $|\psi'\rangle$. Bob can carry out the unitary operation $\sigma_z = |0\rangle\langle 0| - |1\rangle\langle 1|$ on his particle 3. That is

$$\sigma_z|\psi'\rangle = \cos(\theta/2)|0\rangle + \sin(\theta/2)e^{i\varphi}|1\rangle = |\psi\rangle. \tag{10}$$

This means after Bob's unitary operation the state $|\psi\rangle$ has already been prepared in Bob's qubit.

Surely it is possible for Alice to get the state $|\phi_\perp\rangle_1$ after her measurement on particle 1. If so, she will choose another measurement basis $\{|\xi\rangle, |\xi_\perp\rangle\}$ on particle 2, which are written as

$$|\xi\rangle_2 = \frac{1}{\sqrt{2}}(|1\rangle_2 + e^{-i\varphi}|0\rangle_2), \;\; |\xi_\perp\rangle_2 = \frac{1}{\sqrt{2}}(|1\rangle_2 - e^{-i\varphi}|0\rangle_2). \tag{11}$$

Obviously, the basis vectors $\{|\xi\rangle, |\xi_\perp\rangle\}$ and $\{|\eta\rangle, |\eta_\perp\rangle\}$ can be mutually converted by a unitary operation $\sigma_x = |0\rangle\langle 1| + |1\rangle\langle 0|$.

After Alice's measurement, for each collapsed state Bob can employ an appropriate unitary operation to convert it to the prepared state $|\psi\rangle$ except for an overall trivial factor. Here we do not depict them one by one anymore. As a summary, Bob's corresponding unitary operations to Alice's measurement results are listed in Table I. One can easily work out that the total probability of RSP is 1 though the classical communication cost is 2 bits.

By the above analysis, one can easily see that unlike the standard teleportation of an unknown qubit, here, we do not require a Bell-state measurement, which is still more difficult according to the present-day technologies. Only single-particle von Neumann measurement and local operation are necessary. On the other hand, the total

probability of RSP for an arbitrary qubit is 1 while in the previous schemes only the probability of RSP of some special ensembles of qubit is 1. In addition, what deserves mentioning here is that in this protocol, the overall information of the qubit, which is expressed by $\theta$ and $\varphi$, can be divided into two parts. We must first prepare the part $\theta$ and then prepare the remainder part $\varphi$, which can not be transposed. This indicates that the two parts of information are not equal with each other.

As mentioned above, we need only the single-particle measurement and local operation. So, the particle 1 and 2 may be at different locations. In this case, $|\Phi\rangle$ is a real GHZ state. It is natural to generate it to the three-party RSP.

## III. RSP OF HIGHER-DIMENSIONAL QUANTUM STATE FOR THREE PARTIES

In this section, we wish to generalize the RSP protocol to systems with larger than two-dimensional Hilbert space among three parties.

First we consider the case that two parties (Alice and Bob) collaborate with each other to prepare a 4-dimensional quantum state at Charlie's location. A quantum state

$$\begin{aligned}|\psi\rangle = {} & \cos\gamma_1|0\rangle + \sin\gamma_1\cos\gamma_2 e^{i\alpha_1}|1\rangle \\ & + \sin\gamma_1\sin\gamma_2\cos\gamma_3 e^{i\alpha_2}|2\rangle \\ & + \sin\gamma_1\sin\gamma_2\sin\gamma_3 e^{i\alpha_3}|3\rangle \end{aligned} \tag{12}$$

in a four-dimensional Hilbert space can be parameterized by six parameters $\gamma_1, \gamma_2, \gamma_3, \alpha_1, \alpha_2$ and $\alpha_3$ such that $0 \leq \gamma_1, \gamma_2, \gamma_3 \leq \pi/2$ and $0 \leq \alpha_1, \alpha_2, \alpha_3 \leq 2\pi$. Alice and Bob know $\gamma_1, \gamma_2, \gamma_3$ and $\alpha_1, \alpha_2$ and $\alpha_3$ partly respectively, that is, Alice has information of $\gamma_1, \gamma_2, \gamma_3$, and Bob has information $\alpha_1, \alpha_2$ and $\alpha_3$. The quantum channel shared by Alice, Bob and Charlie is a 4-level maximally GHZ state

$$|\Phi\rangle_{ABC} = \frac{1}{2}(|000\rangle + |111\rangle + |222\rangle + |333\rangle)_{ABC}, \tag{13}$$

where particle $A$, $B$ and $C$ belong to Alice, Bob and Charlie respectively. The method is similar to the case of qubit. First Alice must find a set of orthogonal basis vectors to perform a generalized projective measurement on particle $A$. We shall see below, there exist many sets of orthogonal basis vectors that include the state (12). One such set can be obtained by applying a specific unitary transformation on the computational basis vectors

$$\begin{aligned}
U(\gamma_1,\gamma_2,\gamma_3)|0\rangle = |\phi_0\rangle = {} & \cos\gamma_1|0\rangle + \sin\gamma_1\cos\gamma_2|1\rangle \\
& + \sin\gamma_1\sin\gamma_2\cos\gamma_3|2\rangle \\
& + \sin\gamma_1\sin\gamma_2\sin\gamma_3|3\rangle, \\
U(\gamma_1,\gamma_2,\gamma_3)|1\rangle = |\phi_1\rangle = {} & -\sin\gamma_1\cos\gamma_2|0\rangle + \cos\gamma_1|1\rangle \\
& - \sin\gamma_1\sin\gamma_2\sin\gamma_3|2\rangle \\
& + \sin\gamma_1\sin\gamma_2\cos\gamma_3|3\rangle, \\
U(\gamma_1,\gamma_2,\gamma_3)|2\rangle = |\phi_2\rangle = {} & -\sin\gamma_1\sin\gamma_2\cos\gamma_3|0\rangle \\
& + \sin\gamma_1\sin\gamma_2\sin\gamma_3|1\rangle \\
& + \cos\gamma_1|2\rangle - \sin\gamma_1\cos\gamma_2|3\rangle, \\
U(\gamma_1,\gamma_2,\gamma_3)|3\rangle = |\phi_3\rangle = {} & \sin\gamma_1\sin\gamma_2\sin\gamma_3|0\rangle \\
& + \sin\gamma_1\sin\gamma_2\cos\gamma_3|1\rangle \\
& - \sin\gamma_1\cos\gamma_2|2\rangle - \cos\gamma_1|3\rangle.
\end{aligned} \tag{14}$$

Then we have

$$\begin{aligned}|\Phi\rangle_{ABC} = {} & \frac{1}{2}(|\phi_0\rangle_A|\Psi_0\rangle_{BC} + |\phi_1\rangle_A|\Psi_1\rangle_{BC} \\ & + |\phi_2\rangle_A|\Psi_2\rangle_{BC} + |\phi_3\rangle_A|\Psi_3\rangle_{BC}),\end{aligned} \tag{15}$$

where

$$\begin{aligned}
|\Psi_0\rangle_{BC} = {} & \cos\gamma_1|00\rangle + \sin\gamma_1\cos\gamma_2|11\rangle \\
& + \sin\gamma_1\sin\gamma_2\cos\gamma_3|22\rangle \\
& + \sin\gamma_1\sin\gamma_2\sin\gamma_3|33\rangle, \\
|\Psi_1\rangle_{BC} = {} & -\sin\gamma_1\cos\gamma_2|00\rangle + \cos\gamma_1|11\rangle \\
& - \sin\gamma_1\sin\gamma_2\sin\gamma_3|22\rangle \\
& + \sin\gamma_1\sin\gamma_2\cos\gamma_3|33\rangle, \\
|\Psi_2\rangle_{BC} = {} & -\sin\gamma_1\sin\gamma_2\cos\gamma_3|00\rangle \\
& + \sin\gamma_1\sin\gamma_2\sin\gamma_3|11\rangle \\
& + \cos\gamma_1|22\rangle - \sin\gamma_1\cos\gamma_2|33\rangle, \\
|\Psi_3\rangle_{BC} = {} & \sin\gamma_1\sin\gamma_2\sin\gamma_3|00\rangle \\
& + \sin\gamma_1\sin\gamma_2\cos\gamma_3|11\rangle \\
& - \sin\gamma_1\cos\gamma_2|22\rangle - \cos\gamma_1|33\rangle.
\end{aligned} \tag{16}$$

After Alice measures particle $A$, the initial state will be projected onto the measurement basis vectors with the appropriate probability. She has to convey to Bob by classical communication whether to apply the corre-

sponding unitary transformation

$$U_1=\begin{pmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix},$$

$$U_2=\begin{pmatrix}0&0&1&0\\0&0&0&-1\\-1&0&0&0\\0&1&0&0\end{pmatrix},$$

$$U_3=\begin{pmatrix}0&0&0&-1\\0&0&-1&0\\0&1&0&0\\1&0&0&0\end{pmatrix} \tag{17}$$

on his particle $B$ or do nothing. It means that Alice's measurement outcomes $|\phi_0\rangle$, $|\phi_1\rangle$, $|\phi_2\rangle$, and $|\phi_3\rangle$ correspond to unitary transformations $I$, $U_1$, $U_2$, and $U_3$, respectively. Here $I$ is the identity operator.

Next Bob constructs a measurement basis and performs another projective measurement on particle $B$, the relation between the measurement basis $\{\eta_0,\eta_1,\eta_2,\eta_3\}$ and the computational basis $\{|0\rangle,|1\rangle,|2\rangle,|3\rangle$ is given by

$$\begin{aligned}
|\eta_0\rangle &= \frac{1}{2}(|0\rangle+e^{-i\alpha_1}|1\rangle+e^{-i\alpha_2}|2\rangle+e^{-i\alpha_3}|3\rangle),\\
|\eta_1\rangle &= \frac{1}{2}(|0\rangle+ie^{-i\alpha_1}|1\rangle-e^{-i\alpha_2}|2\rangle-ie^{-i\alpha_3}|3\rangle),\\
|\eta_2\rangle &= \frac{1}{2}(|0\rangle-ie^{-i\alpha_1}|1\rangle-e^{-i\alpha_2}|2\rangle+ie^{-i\alpha_3}|3\rangle),\\
|\eta_3\rangle &= \frac{1}{2}(|0\rangle-e^{-i\alpha_1}|1\rangle+e^{-i\alpha_2}|2\rangle-e^{-i\alpha_3}|3\rangle). 
\end{aligned} \tag{18}$$

After Bob measures particle $B$, he will inform Charlie of his measurement result via a classical communication. Charlie can employ an appropriate unitary operation to convert it to the prepared state $|\psi\rangle$. For example, if Alice's measurement result is $|\phi_1\rangle_A$, the state of particle $B$ and $C$, as shown in Eqs. (15) and (16), will collapse into $|\Psi_1\rangle_{BC}$. After Bob receives Alice's measurement result $|\phi_1\rangle_A$, he first carries out the unitary transformation $U_1$ described in Eq.(17) on particle $B$. That is, the unitary operation $U_1$ will transform the state $|\Psi_1\rangle_{BC}$ into

$$\begin{aligned}
|\Psi_1'\rangle = {} & \sin\gamma_1\cos\gamma_2|10\rangle+\cos\gamma_1|01\rangle\\
&+\sin\gamma_1\sin\gamma_2\sin\gamma_3|32\rangle+\sin\gamma_1\sin\gamma_2\cos\gamma_3|23\rangle.
\end{aligned} \tag{19}$$

Next, Bob performs the projective measurement on particle $B$ in the basis described in Eq.(18). According to Bob's different measurement result $|\eta_i\rangle$, Charlie needs to perform the corresponding unitary operation $U_i(C)$ on particle $C$, $U_i(C)$ may take the form of the following $4\times 4$ matrix

$$U_0(C)=\begin{pmatrix}0&1&0&0\\1&0&0&0\\0&0&0&1\\0&0&1&0\end{pmatrix},$$

$$U_1(C)=\begin{pmatrix}0&1&0&0\\i&0&0&0\\0&0&0&-1\\0&0&-i&0\end{pmatrix},$$

$$U_2(C)=\begin{pmatrix}0&1&0&0\\-i&0&0&0\\0&0&0&-1\\0&0&i&0\end{pmatrix},$$

$$U_3(C)=\begin{pmatrix}0&1&0&0\\-1&0&0&0\\0&0&0&1\\0&0&-1&0\end{pmatrix}. \tag{20}$$

TABLE II: Alice's measurement outcome for particle $A$ (AMO), Bob's measurement outcome for particle $B$ (BMO), and Charlie's appropriate unitary operation (CAUO)

| AMO | BMO | CAUO |
|---|---|---|
| $\|\phi_0\rangle_A$ | $\|\eta_0\rangle_B$ | $I$ |
| $\|\phi_0\rangle_A$ | $\|\eta_1\rangle_B$ | $\text{diag}(1,i,-1,-i)$ |
| $\|\phi_0\rangle_A$ | $\|\eta_2\rangle_B$ | $\text{diag}(1,-i,-1,i)$ |
| $\|\phi_0\rangle_A$ | $\|\eta_3\rangle_B$ | $\text{diag}(1,-1,1,-1)$ |
| $\|\phi_2\rangle_A$ | $\|\eta_0\rangle_B$ | $\begin{pmatrix}0&A_1\\A_1&0\end{pmatrix}, A_1=\text{diag}(1,1)$ |
| $\|\phi_2\rangle_A$ | $\|\eta_1\rangle_B$ | $\begin{pmatrix}0&A_2\\-A_2&0\end{pmatrix}, A_2=\text{diag}(1,i)$ |
| $\|\phi_2\rangle_A$ | $\|\eta_2\rangle_B$ | $\begin{pmatrix}0&A_3\\-A_3&0\end{pmatrix}, A_3=\text{diag}(1,-i)$ |
| $\|\phi_2\rangle_A$ | $\|\eta_3\rangle_B$ | $\begin{pmatrix}0&A_4\\A_4&0\end{pmatrix}, A_4=\text{diag}(1,-1)$ |
| $\|\phi_3\rangle_A$ | $\|\eta_0\rangle_B$ | $\begin{pmatrix}0&A_5\\A_5&0\end{pmatrix}, A_5=\begin{pmatrix}0&1\\1&0\end{pmatrix}$ |
| $\|\phi_3\rangle_A$ | $\|\eta_1\rangle_B$ | $\begin{pmatrix}0&A_6\\-A_6&0\end{pmatrix}, A_6=\begin{pmatrix}0&1\\i&0\end{pmatrix}$ |
| $\|\phi_3\rangle_A$ | $\|\eta_2\rangle_B$ | $\begin{pmatrix}0&A_7\\-A_7&0\end{pmatrix}, A_7=\begin{pmatrix}0&1\\-i&0\end{pmatrix}$ |
| $\|\phi_3\rangle_A$ | $\|\eta_3\rangle_B$ | $\begin{pmatrix}0&A_8\\A_8&0\end{pmatrix}, A_8=\begin{pmatrix}0&1\\-1&0\end{pmatrix}$ |

The RSP is completed. Similarly, for other collapsed state corresponding to Alice's measurement result, Bob can employ an appropriate unitary operation in Eq.(17) or do nothing and perform the projective measurement on particle $B$ in the basis described in Eq.(18). Here we do not depict them one by one anymore. As a summary, Bob's measurement outcomes corresponding to Alice's other measurement results, and Charlie's corresponding unitary operations to Bob's measurement results are listed in Table II.

By the above analysis, we may conclude that the essence of our protocol is first preparing a point of the polar circle, and then adding the information of the equatorial state. Now, we suppose that Alice and Bob want

to remotely prepare a known $d$-level quantum state at Charlie's location. However, not all the qudits can be remotely prepared according to Ref. [6], in which the authors have shown that the qudit in real Hilbert space can be remotely prepared when the dimension is 2, 4, or 8. So here we let $d = 8$. The state of an eight-dimensional system can be written as

$$|\psi\rangle = \sum_{i=0}^{7} \cos\theta_i e^{i\varphi_i}|i\rangle, \quad \sum_{i=0}^{7} |\cos\theta_i|^2 = 1. \tag{21}$$

Without loss of generality, we set $\varphi_0 = 0$. According to the analogous procedure described above, the corresponding qudit to be prepared can be remotely prepared exactly onto the particle at Charlie's location. The measurement basis chosen by Alice can be obtained by $V_i|\psi\rangle$. The unitary operation $V_i$ needed is the same as those for eight-dimensional RSP in Ref. [6]. Bob's measurement basis is written as $\{|\eta_j\rangle = \sum_{k=0}^{7} e^{(\pi i/4)jk} e^{i\varphi_j}|k\rangle\}_{j=0}^{7}$.

## IV. CONCLUSIONS

In summary, we have presented a two-step protocol for the exact remote state preparation of an arbitrary qubit using one three-particle GHZ state as the quantum channel. Only a single-particle von Neumann measurement and local operation are necessary. It has been shown that the overall information of the qubit, can be divided into two different parts, which are expressed by $\theta$ and $\varphi$ respectively. We must first prepare the part $\theta$ and then prepare the remainder part $\varphi$, which can not be transposed. This indicates that the two parts of information are not equal with each other. Generalization of this protocol for higher-dimensional Hilbert space systems among three parties is also presented. Moreover, it should be noticed that in this protocol, the information $\theta$ and $\varphi$ may be at different locations. So this protocol may be useful in the quantum information field, such as quantum state sharing, converging the split information at one point, etc. We hope this will provide new insight for investigating more extensive quantum information processing procedures.

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